



## Magneto-thermo nonlinear vibration analysis of pipes reinforced with CNTs

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### ABSTRACT

Nonlinear vibration of embedded polymeric pipes reinforced with single-walled carbon nanotubes (SWCNTs) is investigated in the present work. The classical cylindrical shell theory is used for mathematical modeling of structure. The pipe is subjected to thermal and magnetic loads. The surrounding elastic foundation is simulated with spring constant of Winkler-type and shear constant of Pasternak-type. Mori-Tanaka model is applied in order to obtain the characteristics of the equivalent composite. Based on energy method and Hamilton's principal, the motion equations are derived and solved numerically for calculating the nonlinear frequency utilizing the differential quadrature method (DQM). The effects of different parameters such as volume percent of SWCNTs, geometrical parameters of shell and elastic foundation on the vibration of pipe are investigated. Results showed that with increasing the volume percent of SWCNTs in pipe, the frequency of structure increases.

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### 1. Introduction

Carbon nanotubes (CNTs) possess extraordinary mechanical, thermal and magnetic properties. There have been tremendous research efforts on the potential use of CNT reinforced composites, and progress has been made in various fields (Coleman et al., 2006). New multifunctional materials or "smart materials" have emerged from research labs as a result of the overall development of CNTs composites. Unlike their micron scale counterparts, the physical properties of Nano composites can be altered at an extremely low weight percentage of Nano fillers. While for a given volume fraction, the interfacial regions between nanoparticles and matrix are significantly higher (Ma and Kim, 2011). With the rapid progress of advanced manufacturing techniques, especially the technique of controllable growth of aligned CNTs forest within Nano scale, vertically aligned CNTs arrays can now be fabricated in a large scale at a reasonable low cost. It's been reported that the super-aligned CNT arrays has been produced by continuous spinning of the normal aligned CNTs arrays (Di et al., 2012).

Recently, Shen (2009) incorporated the concept of functionally graded materials (FGM) to the Nano composites by varying the volume fraction of aligned CNTs along the thickness direction of plates. It can be foreseen by this mean the final structural element will have its unique properties that can be tailored in

terms of vibration control, durability, electrical or thermal conductivities. Researches on the bending, buckling, linear and nonlinear vibration of CNT composite shell and beams have appeared by many researchers (Ghorbanpour Arani et al., 2012, 2015; Rafiee et al., 2013; Shen and Xiang 2012) and reported the influence of CNT distributions on the mechanical properties of the structures.

However, it is apparent that no study exists in the open literature on the nonlinear vibration analysis of SWCNT-reinforced composite cylindrical shell based on the classical shell theory and this paper aims to fill in this gap. The mathematic modeling based on the classical shell theory is employed in this study. The geometric nonlinearity of the von Kármán sense is also included. Applying the Euler principle, the virtual energy functional of system is obtained. The DQM is utilized to obtain the eigenvalue equation for the CNT composite pipe. The direct iteration approach is used to search for the nonlinear frequencies at a given vibration amplitude. The influences of SWCNT volume fraction, geometrical parameters, mode numbers and elastic foundation on the nonlinear vibration characteristics of the pipe are discussed.

### 2. Strain-displacement relations

Based on classical shell model, the displacement components are written as:

$$U(x, \theta, z, t) = u(x, \theta, t) - z \frac{\partial w(x, \theta, t)}{\partial x}, \quad (1)$$

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$$V(x, \theta, z, t) = v(x, \theta, t) - z \frac{\partial w(x, \theta, t)}{R \partial \theta}, \quad (2)$$

$$W(x, \theta, z, t) = w(x, \theta, t), \quad (3)$$

Where,  $U$ ,  $V$ ,  $W$  are the displacements of an arbitrary point of the shell in the axial, circumferential and radial directions, respectively,  $u$ ,  $v$ ,  $w$  are the displacements of points on the middle surface of the shell and  $z$  is the distance of the arbitrary point of the shell from the middle surface. Hence, the mechanical strain components can be written as:

$$\varepsilon_{xx} = \varepsilon_{xxm} + zk_{xx}, \quad (4)$$

$$\varepsilon_{\theta\theta} = \varepsilon_{\theta\theta m} + zk_{\theta\theta}, \quad (5)$$

$$\gamma_{x\theta} = \gamma_{x\theta m} + 2zk_{x\theta}, \quad (6)$$

Where the middle surface strains  $\varepsilon_{xxm}$ ,  $\varepsilon_{\theta\theta m}$ ,  $\gamma_{x\theta m}$  and changes in the curvature and torsion of the middle surface  $k_{xx}$ ,  $k_{\theta\theta}$ ,  $k_{x\theta}$  are as follows:

$$\varepsilon_{xxm} = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2, \quad (7)$$

$$\varepsilon_{\theta\theta m} = \frac{\partial v}{R \partial \theta} + \frac{w}{R} + \frac{1}{2} \left( \frac{\partial w}{R \partial \theta} \right)^2, \quad (8)$$

$$\gamma_{x\theta m} = \left( \frac{\partial u}{R \partial \theta} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{R \partial \theta} \right), \quad (9)$$

$$k = \frac{E_m \{ E_m c_m + 2k_r (1 + \nu_m) [1 + c_r (1 - 2\nu_m)] \}}{2(1 + \nu_m) [E_m (1 + c_r - 2\nu_m) + 2c_m k_r (1 - \nu_m - 2\nu_m^2)]}, \quad (14)$$

$$l = \frac{E_m \{ c_m \nu_m [E_m + 2k_r (1 + \nu_m)] + 2c_r l_r (1 - \nu_m^2) \}}{(1 + \nu_m) [E_m (1 + c_r - 2\nu_m) + 2c_m k_r (1 - \nu_m - 2\nu_m^2)]}, \quad (15)$$

$$n = \frac{E_m^2 c_m (1 + c_r - c_m \nu_m) + 2c_m c_r (k_r n_r - l_r^2) (1 + \nu_m)^2 (1 - 2\nu_m)}{(1 + \nu_m) [E_m (1 + c_r - 2\nu_m) + 2c_m k_r (1 - \nu_m - 2\nu_m^2)]} + \frac{E_m [2c_m^2 k_r (1 - \nu_m) + c_r n_r (1 + c_r - 2\nu_m) - 4c_m l_r \nu_m]}{E_m (1 + c_r - 2\nu_m) + 2c_m k_r (1 - \nu_m - 2\nu_m^2)}, \quad (16)$$

$$p = \frac{E_m [E_m c_m + 2p_r (1 + \nu_m) (1 + c_r)]}{2(1 + \nu_m) [E_m (1 + c_r) + 2c_m p_r (1 + \nu_m)]}, \quad (17)$$

$$m = \frac{E_m [E_m c_m + 2m_r (1 + \nu_m) (3 + c_r - 4\nu_m)]}{2(1 + \nu_m) \{ E_m [c_m + 4c_r (1 - \nu_m)] + 2c_m m_r (3 - \nu_m - 4\nu_m^2) \}}, \quad (18)$$

Where  $k_r$ ,  $l_r$ ,  $m_r$ ,  $n_r$ ,  $p_r$  can be obtained in (Coleman et al., 2006) and  $C_r$  is also the volume present of the reinforced SWCNTs in matrix.

#### 4. Energy method

$$k_{xx} = -\frac{\partial^2 w}{\partial x^2}, \quad (10)$$

$$k_{\theta\theta} = -\frac{1}{R^2} \frac{\partial^2 w}{\partial \theta^2}, \quad (11)$$

$$k_{x\theta} = -\frac{\partial^2 w}{R \partial x \partial \theta}. \quad (12)$$

#### 3. Stress-strain relations

Based on classical cylindrical shell theory, the stress-strain relations may be expressed as:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \tau_{x\theta} \end{bmatrix} = \begin{bmatrix} \underbrace{C_{11}}_{k+m} & \underbrace{C_{12}}_l & 0 \\ \underbrace{C_{21}}_l & \underbrace{C_{22}}_n & 0 \\ 0 & 0 & \underbrace{C_{66}}_p \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} - \alpha_{xx} \Delta T \\ \varepsilon_{\theta\theta} - \alpha_{\theta\theta} \Delta T \\ \gamma_{x\theta} \end{bmatrix}, \quad (13)$$

Where  $C_{ij}$  and  $\alpha_{ii}$  ( $i = x, \theta$ ) are elastic constants and thermal expansion, respectively. Furthermore,  $k, l, m, n, p$  are Hill's constant which may be calculated by Mori-Tanaka model (Ghorbanpour Arani et al., 2012) as follows:

The total potential energy of the pipe is the sum of strain energy  $U$ , kinetic energy  $K$ , and the work  $W$  done by the applied load. The strain energy is:

$$U = \frac{1}{2} \iiint (\sigma_{xx} \varepsilon_{xx} + \sigma_{\theta\theta} \varepsilon_{\theta\theta} + \tau_{x\theta} \gamma_{x\theta}) R dx d\theta dz. \quad (19)$$

Strain energy by combining Eq. 18 and Eq. 19, may be written as:

$$U = \frac{1}{2} \int_{-h/2}^{h/2} \int_0^{2\pi} \int_0^L \left\{ \sigma_{xx} \left[ \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 - z \frac{\partial^2 w}{\partial x^2} \right] + \sigma_{\theta\theta} \left[ \frac{\partial v}{R \partial \theta} + \frac{w}{R} + \frac{1}{2} \left( \frac{\partial w}{R \partial \theta} \right)^2 - \frac{z}{R^2} \frac{\partial^2 w}{\partial \theta^2} \right] + \tau_{x\theta} \left[ \frac{\partial u}{R \partial \theta} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{R \partial \theta} - 2z \frac{\partial^2 w}{R \partial x \partial \theta} \right] \right\} R dx d\theta dz. \quad (20)$$

The kinetic energy of system may be written as:

$$K = \frac{\rho}{2} \iiint (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) R dx d\theta dz \quad (21)$$

The work done by applied force by elastic foundation and magnetic field can be expressed as: (Ghorbanpour Arani et al., 2015)

$$W = \int (F_e w) dA = \int \left( (k_w w - k_g \nabla^2 w) + \eta H_x^2 \frac{\partial^2 w}{\partial x^2} \right) w dA, \quad (22)$$

Where  $k_w, k_g, \eta$  and  $H_x$  are spring constant of elastic medium, shear constant of elastic medium, magnetic permeability and magnetic field, respectively? Applying Euler principle as follows:

$$\begin{cases} \frac{\partial F}{\partial u} - \frac{\partial}{\partial x} \left( \frac{\partial F}{\partial u_{,x}} \right) - \frac{\partial}{\partial \theta} \left( \frac{\partial F}{\partial u_{,\theta}} \right) = \rho h \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial F}{\partial v} - \frac{\partial}{\partial x} \left( \frac{\partial F}{\partial v_{,x}} \right) - \frac{\partial}{\partial \theta} \left( \frac{\partial F}{\partial v_{,\theta}} \right) = \rho h \frac{\partial^2 v}{\partial t^2}, \\ \frac{\partial F}{\partial w} - \frac{\partial}{\partial x} \left( \frac{\partial F}{\partial w_{,x}} \right) - \frac{\partial}{\partial \theta} \left( \frac{\partial F}{\partial w_{,\theta}} \right) + \frac{\partial^2}{\partial x^2} \left( \frac{\partial F}{\partial w_{,xx}} \right) \\ + \frac{\partial^2}{\partial x \partial \theta} \left( \frac{\partial F}{\partial w_{,x\theta}} \right) + \frac{\partial^2}{\partial \theta^2} \left( \frac{\partial F}{\partial w_{,\theta\theta}} \right) = \rho h \frac{\partial^2 w}{\partial t^2}. \end{cases} \quad (23)$$

The motion equations can be written as:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{x\theta}}{R \partial \theta} = \rho h \frac{\partial^2 u}{\partial t^2}, \quad (24)$$

$$\frac{\partial N_\theta}{R \partial \theta} + \frac{\partial N_{x\theta}}{\partial x} = \rho h \frac{\partial^2 v}{\partial t^2}, \quad (25)$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{2 \partial^2 M_{x\theta}}{R \partial x \partial \theta} + \frac{\partial^2 M_\theta}{R^2 \partial \theta^2} - \frac{N_\theta}{R} + N_x \frac{\partial^2 w}{\partial x^2} + N_\theta \frac{\partial^2 w}{R^2 \partial \theta^2} + N_{x\theta} \frac{2 \partial^2 w}{R \partial x \partial \theta} - F_e = \rho h \frac{\partial^2 w}{\partial t^2}, \quad (26)$$

Where the internal forces and moments may be expressed as

$$\begin{cases} N_x \\ N_\theta \\ N_{x\theta} \end{cases} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{cases} \sigma_x \\ \sigma_\theta \\ \tau_{x\theta} \end{cases} dz, \quad (27)$$

Substitute Eq. 13 into Eqs. 27 and 28 yields:

$$N_x = \left( h C_{11} \left( \frac{\partial u}{\partial x} + 0.5 \left( \frac{\partial w}{\partial x} \right)^2 - \alpha_x \Delta T \right) + C_{12} \left( \frac{\partial v}{R \partial \theta} + \frac{w}{R} + 0.5 \left( \frac{\partial w}{R^2 \partial \theta} \right)^2 - \alpha_\theta \Delta T \right) \right), \quad (29)$$

$$N_\theta = \left( h C_{12} \left( \frac{\partial u}{\partial x} + 0.5 \left( \frac{\partial w}{\partial x} \right)^2 - \alpha_x \Delta T \right) + C_{22} \left( \frac{\partial v}{R \partial \theta} + \frac{w}{R} + 0.5 \left( \frac{\partial w}{R^2 \partial \theta} \right)^2 - \alpha_\theta \Delta T \right) \right), \quad (30)$$

$$N_{x\theta} = h \left( C_{66} \left( \frac{\partial u}{R \partial \theta} + \frac{\partial v}{\partial x} + \frac{\partial w}{R \partial \theta} \frac{\partial w}{\partial x} \right) \right), \quad (31)$$

$$M_x = \frac{h^3}{12} \left( C_{11} \left( -z \frac{\partial^2 w}{\partial x^2} \right) + C_{12} \left( -z \frac{\partial^2 w}{R^2 \partial \theta^2} \right) \right), \quad (32)$$

$$M_\theta = \frac{h^3}{12} \left( C_{12} \left( -z \frac{\partial^2 w}{\partial x^2} \right) + C_{22} \left( -z \frac{\partial^2 w}{R^2 \partial \theta^2} \right) \right), \quad (33)$$

$$M_{x\theta} = \frac{h^3}{12} C_{66} \left( -2z \frac{\partial^2 w}{R \partial \theta \partial x} \right), \quad (34)$$

Defining dimensionless parameters:

$$\begin{aligned} \gamma = \frac{h}{L}, \quad \xi = \frac{x}{L}, \quad \beta = \frac{h}{R}, \quad \{\bar{u}, \bar{v}, \bar{w}\} = \frac{\{u, v, w\}}{h}, \quad \bar{C}_{ij} = \frac{C_{ij}}{C_{11}}, \\ K_w = \frac{h k_w}{C_{11}}, \quad K_g = \frac{k_g}{h C_{11}}, \quad \bar{t} = \frac{t}{h \sqrt{\frac{\rho_f}{C_{s11}}}}, \quad \bar{H}_x = \frac{\eta H_x^2}{C_{11}} \end{aligned} \quad (35)$$

Substitute Eqs. 29- 35 into Eqs. 24-26 and using above relation results:

$$(\gamma^2) \left( \frac{\partial^2 \bar{u}}{\partial \xi^2} + \frac{\partial \bar{w}}{\partial \xi} \frac{\partial^2 \bar{w}}{\partial \xi^2} \right) + \gamma \beta \bar{C}_{12} \left( \frac{\partial^2 \bar{v}}{\partial \xi \partial \theta} + \frac{\partial \bar{w}}{\partial \xi} + \beta \frac{\partial \bar{w}}{\partial \theta} \frac{\partial^2 \bar{w}}{\partial \xi \partial \theta} \right) + \beta \bar{C}_{66} \left( \beta \frac{\partial^2 \bar{u}}{\partial \theta^2} + \gamma \frac{\partial^2 \bar{v}}{\partial \xi \partial \theta} + \beta \frac{\partial \bar{w}}{\partial \xi} \frac{\partial^2 \bar{w}}{\partial \theta^2} + \gamma \frac{\partial \bar{w}}{\partial \xi} + \beta \frac{\partial \bar{w}}{\partial \theta} \frac{\partial^2 \bar{w}}{\partial \xi \partial \theta} \right) = \frac{\partial^2 \bar{u}}{\partial \bar{t}^2} \quad (36)$$

$$\beta \bar{C}_{12} \left( \gamma \frac{\partial^2 \bar{u}}{\partial \xi \partial \theta} + \gamma^2 \frac{\partial \bar{w}}{\partial \xi} \frac{\partial^2 \bar{w}}{\partial \xi \partial \theta} \right) + \beta^2 \bar{C}_{22} \left( \frac{\partial^2 \bar{v}}{\partial \theta^2} + \frac{\partial \bar{w}}{\partial \theta} + \beta \frac{\partial \bar{w}}{\partial \theta} \frac{\partial^2 \bar{w}}{\partial \theta^2} \right) + \gamma \bar{C}_{66} \left( \beta \frac{\partial^2 \bar{u}}{\partial \xi \partial \theta} + \gamma \frac{\partial^2 \bar{v}}{\partial \xi^2} + \beta \gamma \frac{\partial^2 \bar{w}}{\partial \theta \partial \xi} \frac{\partial \bar{w}}{\partial \xi} + \beta \gamma \frac{\partial \bar{w}}{\partial \theta} \frac{\partial^2 \bar{w}}{\partial \xi^2} \right) = \frac{\partial^2 \bar{v}}{\partial \bar{t}^2} \quad (37)$$

$$\frac{\gamma^2}{12} \left( -\gamma^2 \frac{\partial^4 \bar{w}}{\partial \xi^4} - \bar{C}_{12} \beta^2 \frac{\partial^4 \bar{w}}{\partial \xi^2 \partial \theta^2} \right) - \frac{\gamma^2 \beta^2 \bar{C}_{66}}{3} \left( \frac{\partial^4 \bar{w}}{\partial \xi^2 \partial \theta^2} \right) + \frac{\beta^2}{12} \left( -\gamma^2 \bar{C}_{12} \frac{\partial^4 \bar{w}}{\partial \theta^4} - \bar{C}_{66} \beta^2 \frac{\partial^4 \bar{w}}{\partial \xi^2 \partial \theta^2} \right) - \gamma \beta \bar{C}_{12} \left( \frac{\partial \bar{u}}{\partial \xi} + \frac{\gamma}{2} \left( \frac{\partial \bar{w}}{\partial \xi} \right)^2 \right) - \beta \bar{C}_{22} \left( \beta \frac{\partial \bar{v}}{\partial \theta} + \beta \bar{w} + \frac{\beta^2}{2} \left( \frac{\partial \bar{w}}{\partial \theta} \right)^2 \right) - \left( \beta^2 \bar{C}_{12} \alpha_x + \beta^2 \bar{C}_{22} \alpha_\theta \right) \frac{\partial^2 \bar{w}}{\partial \theta^2} \Delta T$$

$$- \left( \gamma^2 \alpha_x + \gamma^2 \bar{C}_{12} \alpha_\theta \right) \frac{\partial^2 \bar{w}}{\partial \xi^2} \Delta T - (K_w) \bar{w} + (K_g) \left( \gamma \frac{\partial^2 \bar{w}}{\partial \xi^2} + \beta \frac{\partial^2 \bar{w}}{\partial \theta^2} \right) - \bar{H}_x \frac{\partial^2 \bar{w}}{\partial \xi^2} = \frac{\partial^2 \bar{w}}{\partial \bar{t}^2}.$$

## 5. DQM

DQM is employed which in essence approximates the partial derivative of a function, with respect to a spatial variable at a given discrete point, as a weighted linear sum of the function values at all discrete points chosen in the solution domain of the spatial variable. Let  $F$  be a function representing  $u$ ,  $v$ ,  $w$  and  $\phi$  with respect to variables  $x$  and  $\theta$  in the

$$\frac{d^n f_x(x_i, \theta_j)}{dx^n} = \sum_{k=1}^{N_x} A_{ik}^{(n)} f(x_k, \theta_j) \quad n = 1, \dots, N_x - 1. \quad (39)$$

$$\frac{d^m f_y(x_i, \theta_j)}{d\theta^m} = \sum_{l=1}^{N_\theta} B_{jl}^{(m)} f(x_i, \theta_l) \quad m = 1, \dots, N_\theta - 1. \quad (40)$$

$$\frac{d^{n+m} f_{xy}(x_i, \theta_j)}{dx^n d\theta^m} = \sum_{k=1}^{N_x} \sum_{l=1}^{N_\theta} A_{ik}^{(n)} B_{jl}^{(m)} f(x_k, \theta_l). \quad (41)$$

Where  $A_{ik}^{(n)}$  and  $B_{jl}^{(m)}$  are the weighting coefficients;  $N_x$  and  $N_\theta$  are grid points which may be found in (Ghorbanpour Arani et al. 2015). According

following domain of ( $0 < x < L$ ,  $0 < \theta < 2\pi$ ) having  $N_x \times N_\theta$  grid points along these variables. The  $n^{\text{th}}$ -order partial derivative of  $F(x, \theta)$  with respect to  $x$ , the  $m^{\text{th}}$ -order partial derivative of  $F(x, \theta)$  with respect to  $\theta$  and the  $(n + m)^{\text{th}}$ -order partial derivative of  $F(x, \theta)$  with respect to both  $x$  and  $\theta$  may be expressed discretely at the point  $(x_i, \theta_i)$  as:

to DQM, mechanical boundary conditions may be written as:

$$\begin{cases} w_{i1} = v_{i1} = u_{i1} = 0, & \sum_{j=1}^{N_\theta} A_{2j} w_{ji} = 0 \\ w_{N_x i} = v_{N_x i} = u_{N_x i} = 0, & \sum_{j=1}^{N_\theta} A_{(N_x-1)j} w_{ji} = 0 \end{cases} \quad \text{for } i = 1 \dots N_\theta \quad (42)$$

Applying these boundary conditions into the governing yields the following coupled assembled matrix equations:

$$\left( \left[ \frac{K_L + K_{NL}}{K} \right] + \Omega^2 [M] \right) \begin{Bmatrix} \{d_b\} \\ \{d_d\} \end{Bmatrix} = 0, \quad (43)$$

$$\begin{aligned} \{d_b\} &= \{ \bar{u}_i, \bar{v}_{i1}, \bar{w}_{i1}, \bar{w}_{i2}, \Phi_{i1}, \bar{u}_{iN_\theta}, \bar{v}_{iN_\theta}, \bar{w}_{iN_\theta}, \bar{w}_{i(N_\theta-1)}, \Phi_{iN_\theta} \} \quad i = 1, \dots, N_x \\ \{d_d\} &= \{ \bar{u}_{ij}, \bar{v}_{ij}, \bar{w}_{i(j+1)}, \Phi_{ij} \} \quad i = 1, \dots, N_x, j = 2, \dots, N_x - 1 \end{aligned} \quad (44)$$

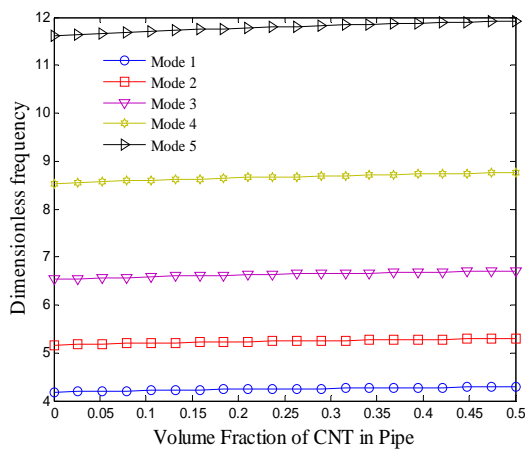
Finally, based on an iterative method and eigenvalue problem, the frequency of structure may be obtained.

## 6. Numerical result

In order to obtain the frequency for considered pipe embedded in the Pasternak foundation, DQM was used in conjunction with a program being written in MATLAB, where the effect of volume percent of SWCNTs, mode numbers, geometrical parameters and elastic medium were investigated.

### 6.1. The effects of different parameters

The effect of mode numbers on the nonlinear frequency of pipe versus volume percent of SWCNTs is showed in Fig. 1. As can be seen, with increasing the mode numbers, the nonlinear frequency increases.

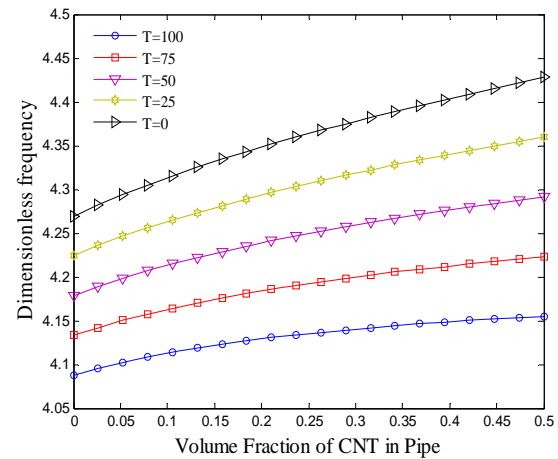


**Fig. 1:** The effect of mode numbers on the nonlinear frequency against volume percent of SWCNTs

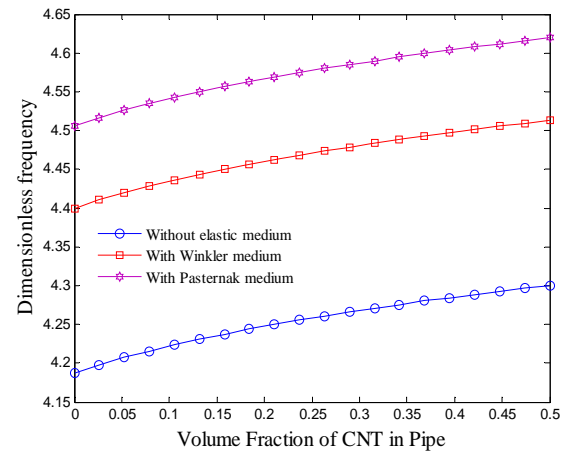
The effect of volume percent of SWCNTs and temperature gradient on the nonlinear frequency is illustrated in Fig. 2. It can be found that with increasing the volume percent of SWCNTs, the nonlinear frequency increases. It is due to the fact that with increasing volume percent of SWCNTs in pipe, the stiffness of structure increases. Hence, the SWCNT volume fraction is effective controlling parameters for vibration of the pipes. In addition, the nonlinear frequency is decreased with increasing temperature gradient due to reduction in stiffness of structure.

Fig. 3 illustrates the influence of elastic medium, including Winkler and Pasternak modules, on the frequency, along the volume percent of SWCNTs. Obviously, the elastic medium type has a significant effect on vibration of the pipe, since the frequency of the system in the case of without elastic medium are lower than other cases. It can be concluded that the frequency for Pasternak model is higher than Winkler one. The above results are reasonable, since the Pasternak medium considers not only the normal stresses (i.e. Winkler foundation) but also the

transverse shear deformation and continuity among the spring elements.

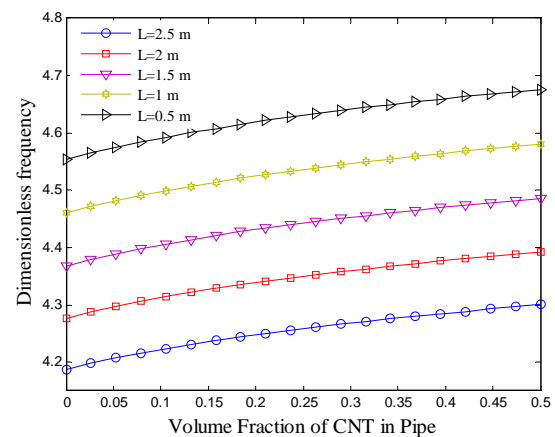


**Fig. 2:** The effect of temperature gradient on the nonlinear frequency against volume percent of SWCNTs

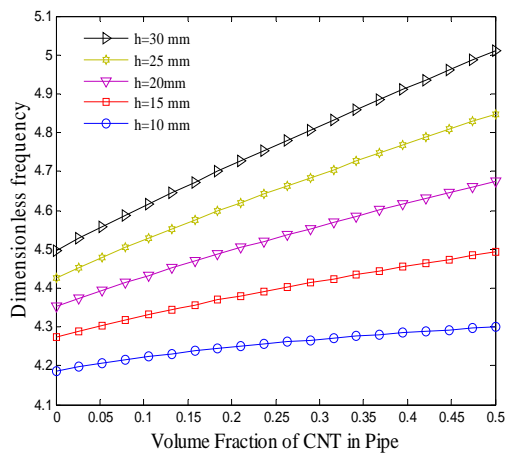


**Fig. 3:** The effect of elastic foundation on the nonlinear frequency against volume percent of SWCNTs

Figs. 4 and 5 demonstrate the influence of geometrical parameters on the nonlinear frequency versus volume percent of SWCNTs. As can be seen, with increasing thickness of pipe and decreasing pipe length, the nonlinear frequency is increased. It is physically reasonable, since with increasing thickness and decreasing length of pipe, the stiffness of structure increases.



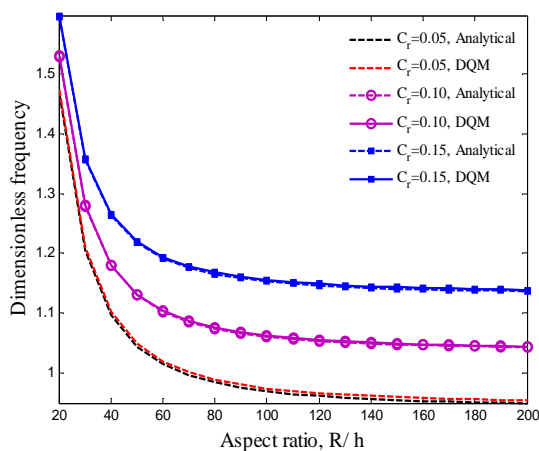
**Fig. 4:** The effect of pipe length on the nonlinear against volume percent of SWCNTs



**Fig. 5:** The effect of pipe thickness on the nonlinear against volume present of SWCNTs

## 6.2. Validation

In order to validate the results of present work, neglecting the nonlinear terms in motion equations, the frequency of structure is calculated with Navier method and compared with the results of DQM. As can be seen from Fig. 6, which shows the dimensionless frequency versus radius to thickness ratio for different volume percent of SWCNTs, illustrates the accuracy of present results, indicating validation of this work.



**Fig. 6:** Validation of present work

## 7. Conclusion

Present work deals with thermo-magneto nonlinear vibration analysis of embedded polymeric pipes reinforced with SWCNTs resting on elastic medium. The pipe and elastic medium were simulated with classical cylindrical shell and Pasternak foundation, respectively. Based on energy method and Euler principal, the motion equations were derived. DQM was applied for obtaining the nonlinear frequency of structure. The numerical results indicate that with increasing volume percent of SWCNTs in pipe, the frequency was increased. Furthermore, considering elastic foundation increases the frequency of system. In addition, with increasing thickness of pipe and decreasing pipe

length, the nonlinear frequency is increased. The results of this study are validated as far as possible by comparing the analytical and numerical methods in evaluating the nonlinear frequency of pipe. Finally, it is hoped that the obtained results might be useful for the design and improvement of industrial pipes applying nanotechnology.

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